INERTIAL MECHANICS PERSPECTIVE ON FLEXIBLE TURBINE-GENERATOR ROTOR VIBRATIONS, DIAGNOSTICS AND BALANCING

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Abstract: This paper looks at the effects of a rotor's inertia and momentum and its interrelation with vibration diagnostics and rotor balancing, and the link between theory and practice. Of particular focus regarding both balancing and field diagnostics is the effect of distributed mass eccentricities of a continuous rotor, and associated bearing stability of rotor journals, and the operating orientation of the rotor body governed by momentum and inertia. These effects vary relative to rotor speed, with bearing-journal stability being gravity-governed in the sub-harmonic and trans-harmonic frequency regions, versus rotor behavior and stability governed by rotor inertia and momentum at super-critical and operating frequency regions, particularly rotor mass axis self-centering within bearing clearances. These effects also vary in proportion to applied load torque at constant speed. In field vibration diagnostics, the root cause of many apparent vibration problems is in the imposed constraints that counter the natural inertia-governed tendency of motion of the rotor, where bearings or adjacent coupled rotors restrict the rotor's natural operating orientation. Dynamic vibration problems and what appears as resonant excitation are in many cases only a consequence of unresolved static problems, such as rotor mass axis eccentricity or a bow. Shop balancing should focus on resolving and compensating mass axis eccentricity at speeds within the rotor fundamental harmonic resonance frequency range, using three correction planes, maintaining mass symmetry at any sub-critical or super-critical frequency.

Keywords: Balancing; Critical speed; Eccentricity; Harmonics; Inertia; Resonance; Quasi-High speed balancing; Rotor Stability

Introduction:

In the area of vibration diagnostics and practical balancing of large turbines and generator rotors, diagnosticians and balancers alike focus almost exclusively on observing and recording relative “vibration displacements”, specifically the symptoms of dynamic motion from centrifugal forces, system resonant conditions, and various system dynamic parameters. However, the more fundamentally important, but mostly neglected root cause in diagnosing rotor vibration displacements is the lateral, pseudo-static stability of the rotor, as governed by its rotational inertia and momentum.

A rotor's natural stability orientation is best recognized via its non-rotating (inertial) reference frame that is established in the interaction between rotor lateral motion and bearing oil hydrodynamic forces, which for horizontal machines must also take into account the gravitational environment. This is reflected in each journal's attitude angle in each respective bearing. It is important to note that through the rotor's full speed range, this "pseudo-static" inertial reference frame, or spatial reference of the center of rotation, does not remain at in a constant position within the free space of the bearing clearance, nor does it remain referenced to a constant location on the rotor itself through the entire speed range. The notion of "static" or "pseudo-static" used here refers to the axis formed by shaft centerline location of all radial planes of the continuous rotor, representing the position in space of the "static" center of rotation.

The lateral pseudo static stability of the non-rotating (inertial) reference frame is established between rotor lateral motion and bearing oil hydrodynamic forces in a gravitational environment. This equilibrium is established by the cross-coupling between forces within the rotor generated and transformed from input torque (viewed in a rotating reference frame), and the forces generated in the hydrodynamic bearings and supports [9] as shown in Figure 1. The rotor journal maintains a pseudo-static position in space at an attitude angle of the apparent gravity force, as established between oil pressure acting against gravity load and the lateral translation of the rotor mass axis, and oil reaction force flowing through the orifice established between the journal and bearing.

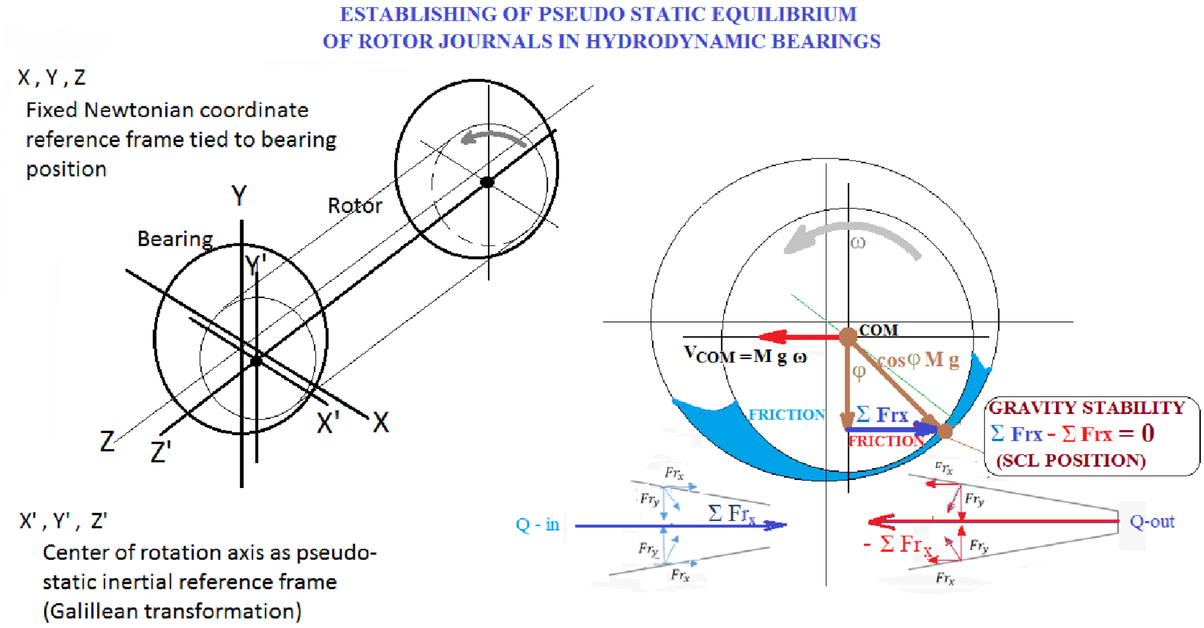


Figure 1. Pseudo-static stable position of the rotor journal in hydrodynamic bearings.

It must also be recognized that the "static" non-rotating reference frame changes its position relative to the solid rotor itself while crossing the first system fundamental harmonic resonance frequency range (or “1st critical speed”). This "static" reference frame initially at standstill (in Newtonian coordinates) matches the rotor's journal centerline axis, and in space correlates to the rotor journals sitting on the bottom of the bearings. With initial drive torque input and acceleration of the continuous rotor, this reference frame of static stability slowly moves in space within the bearing clearance, shown as the shaft centerline (SCL) path, but while remaining tied to the journal centerline as being the center of rotation (stable in a Galilean transformation frame). While following the SCL path, upon reaching the first system critical speed peak at a phase angle of ~ 90°, the rotational momentum of the rotor becomes sufficiently large and begins to govern the natural stability orientation of the rotor, and likewise drives the rotor to self-center its rotation about its mean center-of-mass axis in "natural" motion.

If there is minimal mass eccentricity in the rotor, meaning minimal distance and/or skew between the rotor's journal centerline and mean mass centerline, then no real change occurs in the "static" reference frame, relative to the rotor. Likewise, no dynamic radial response will be seen, and such a rotor will be “balanced” at any speed maintaining symmetry in any reference frame [12]. However, if mass eccentricity is measurable, then upon reaching the first system critical speed and above a phase angle of 90°, this static reference frame will be defined by the rotor's overall mass axis, while the journal centerline (or what was the initial "static" reference frame on the rotor) will now begin to whirl around it. This corresponds to the total 180° phase shift seen through the critical speed region, and represents the condition for speeds above this point.

This process is more easily observed and visualized when considering a vertical rotor, without the effect of gravity holding journals in bearings. In a vertical rotor-bearing system with no bearing gravity loading, the effective combined system stiffness of the rotors plus bearings is zero, and therefore the *system* critical speed is zero. (The rotor of course still has its own inherent independent harmonic resonance frequency.) This means that immediately upon initiating rotation, the rotor will naturally rotate as a rigid body about its mean center of mass axis in what can be considered "natural" centroidal rotation, following the conservation of angular momentum. A horizontal rotor has the same innate tendency, but due to gravity pressing the journals into the bearings, it is held to rotate about its journal axis regardless of rotor body eccentricity until reaching the peak of the “first system critical speed”, gaining sufficient speed and rotational momentum to initiate this centroidal rotation about the mass axis, which can then be taken as the new "static" stability reference frame center about which displacement measurements are effectively read.

Applied to rotor balancing on a high-speed shop balancing machine, it then becomes crucial to pay attention to what reference frame the vibration displacement readings are being measured against, and this comes down to recognizing the speed region, relative to the rotor-bearing system's first critical speed, or first system fundamental harmonic resonance frequency. Vibration displacement measurements taken at sub-harmonic speeds will indicate motion relative to the journal axis centerline. At super-harmonic speeds, displacement measurements are taken relative to the true mass centerline, but while also incorporating the rotor body's natural shift in shaft centerline position in “space” as affected by the rotor's momentum, which can alter the measured relative vibration displacement depending on the reference frame location that the center of rotation is tied to.

With regard to rotor balancing on a balancing machine, a rotor being balanced at super-critical speeds will result in the balancing process balancing the rotor about its mass axis, with the journals free to adjust themselves to a position of unconstrained natural "static" stability. For instance, on a low-speed, soft-bearing balancing machine with low system stiffness, all balancing of displacement responses will be performed about and relative to the rotors mean mass axis. On a hard-bearing or high-speed machine, the same applies, but would be the case only at higher speeds above the corresponding system fundamental harmonic resonance frequency (1st critical), while balancing at sub-harmonic frequencies on such a machine would be performed about and relative to the rotor's journal axis.

If applying traditional shop balancing methods on an eccentric rotor, a residual mass axis skew might still exist unresolved at full operating speed despite low measured shop vibration displacement and bearing seismic readings. In such cases, these low dynamic readings are often at the expense of improper SCL position or excessive shifting and SCL path hysteresis, which indicate the natural inertial self-alignment tendency governed by the rotor's mass axis from unresolved eccentricity. Reviewing SCL readings should be made an important part of any rotor balancing procedure, as it provides crucial indications of unresolved mass eccentricity that may create vibration problems once installed in the field.

When installed and assembled in the field, a rotor will encounter physical constraints (such as being coupled to adjacent rotor mass), preventing its independent "natural" shaft centerline adjustment to its mass axis if there is any residual mass axis skew, and the rotor will be constrained to rotate about its journal axis. Any constraints against the natural inertial tendency of the rotor will create reaction forces, which are seen as measured vibration. Therefore, the full shop rotor balancing process should be performed primarily up to the first critical speed when still referenced to the journal axis, such that the balance condition achieved on the balancing machine matches the later forced operating orientation of the installed rotor.

1. Terminology and Categorization:

Since the intent of this paper is to bring together science and practical technology in the area of vibration diagnostics and balancing of large rotating machinery, it is necessary to broaden the standard vocabulary currently in use with some expanded definitions to add clarity to the later descriptions of rotor behavior.

1.1 Unbalance and balancing:

1.1.1 What is rotor “unbalance”?

The initial assumptions are of dealing with a continuous, large, “rigid” or flexible rotor in null-mode (its intrinsic shape at standstill), driven in circular motion by externally applied torque, with the rotor's longitudinal axis horizontally oriented perpendicular to gravity, and constrained at two flexible, fluid-film bearing supports. Rotor vibration in practice is most commonly attributed to "unbalance". However, the notion of "unbalance" of continuous rotors is usually not simply the effect of centrifugal force from a radially asymmetric lumped mass excess or deficiency in a single random radial plane, but rather is usually in the form of axially distributed mass axis eccentricities or runouts between supports (gravity constraints), such as a shaft bow. This causes the real longitudinal center-of-mass axis of the rotor (its true centroidal axis) to not be coincident with its geometrically-centered, forced axis of rotation, defined as the axis connecting the journal centers.

Due to the rotor journals being held and constrained in the bearings by gravity, the rotor mass axis is forced to precess about this journal axis (the orthogonal Z-axis, if a radial plane is taken as X-Y). Gravity prevents the forward momentum gained from drive torque from bringing the rotor to spin in its natural centroidal circular motion, as it otherwise would if unconstrained or unaffected by gravity.

To better define rotor “unbalance”, a rotor can be conceptually divided into fundamental harmonic modal elements corresponding to the segments of the rotor that remain "rigid" during modal deflection of a flexible rotor. The minimum number of nodes of these modal elements fits the formula 2N+1, where N is the number of the mode. For the first mode, this corresponds to two elements, each comprising one half of the rotor, meeting at the plane of the total rotor's axial center of mass (COM), and thereby having 3 node points. When mass axis of a continuous rotor is radially offset and skewed between constraints, the infinite number of axially distributed radial mass eccentricities can be reduced to only three equivalent particle point masses, at the total rotor COM and at the two COMs of the two modal elements of the rotor’s fundamental harmonic resonance frequency mode. **These three simultaneous distributed COMs are the absolute minimum number of locations for the effective simultaneous superposition of all mass unbalances or eccentricities on a flexible rotor**, for the purpose of calculating or modeling the solution to unbalance or eccentricity. This concept of modal elements becomes particularly relevant for the optimal practical balancing method for flexible, eccentric or bowed rotors.

Rotor “unbalance” is therefore a state when a longitudinal line which intersects the COM of the whole rotor and centers of masses of the fundamental harmonic mode modal elements (this line defines the rotor’s centroidal mass axis), is radially offset and not coincident with its geometrically-centered, forced axis of rotation, defined as the axis connecting the journal centers (the non-centroidal axis).

1.1.2 What is rotor “balancing”?

In order to avoid any amplified response at the “1st system critical”, the balancing of large turbine and generator rotors in horizontal machines should be the process of “aligning” the rotor’s net principal inertia axis (the rotor's “rigid body” mass axis between its bearing constraints in a state of rest), to coincide with the rotor’s gravity-constrained axis of rotation (journal axis) in any reference frame, independent of spatial orientation or rotation velocity or acceleration. Superimposing all unbalance/eccentricity into a single equivalent total at a point at the total rotor COM (as is commonly done in rotor modeling and for standard balancing methods) is not a correct approach when dealing with eccentric flexible rotors. True mechanical rotor “balancing” to achieve a balanced (or better stated "aligned and compensated") condition should be generally done on balancing machines, by placement of correction weights proportionally axially distributed simultaneously in a minimum of three axially predetermined balancing planes to compensate the effect of centrifugal forces from these internal three concentrated COMs.

The sum of radial forces and axial moments produced by the added weights should create a virtual dynamic mass axis that should as close as possible mirror the eccentricity of the inherent mass axis at any speed, or to at least within the allowable eccentricity per the standards given in ISO 1940-1 [1]. The geometric sum of the intrinsic mass axis and the "virtual" mass axis created by the balance weights should ideally result in a vanishing of synchronous cyclic reaction forces and displacements at the points of constraint (journals) of the rotor, and rotor will be balanced at any speed in any reference frame.

1.2. Dynamic - Orbital motion:

A rotor as a continuous body in a rotor-bearing system is utilized as device to transfer circular rotating motion to work, as an open system with continuous energy input (in the form of applied torque). There are several types of rotating motions in different reference frames that a rotor exhibits through its full speed range, with distinct rotor behavior within various operating speed regions. These speed regions include accelerating from the state of rest through sub-harmonic speeds to the beginning of the first system fundamental harmonic speed range (first system critical speed range), specifically passing through the first system fundamental harmonic speed range (trans-harmonic), and from the start of super-harmonic speeds to design operating speed frequency.

1.2.1 Spin:

Spin is a purely synchronous circular motion of the continuous rotor mass initiated by external force (applied torque) about the rotor's neutral axis, generally corresponding to the rotor journal axis. All internal particle masses of the rotor, in each infinitesimal axial "slice" of a continuous rotor, are orbiting synchronously at constant radial distance around the neutral axis. Only with a perfectly concentric rotor would spin be the only observed motion, and would be maintained without any further net effect on rotor internal mass from external forces. In pure spin, with the rotor unconstrained, the momentum, angular momentum and energy would be conserved.

In general practice, rotor spin is maintained about the neutral axis of applied torque, but is also combined with other modes of rotation (orbital translation, precession). The journals (rotating and gravity-constrained in oil lubricated bearings) link rotation and translation of the mass axis, establishing a statically stable equilibrium within span **L** between support constraints.

1.2.2 Synchronous lateral orbital translation of mass axis (or precession):

Lateral orbital translation (or shaft orbit) is the circular (but not necessarily rotating) path in space generally synchronous to each revolution. At sub-harmonic speeds, this correlates to the motion of the mass axis of the rotor translating (precessing) [6, 10] internally within the rotor about the journal axis, if one were to trace out its path in space. In the fundamental system harmonic speed range, this path of the mass axis will grow in radius and change shape as the rotor is accelerated, up to ~ 90 degrees relative phase angle, as the rotor bends and deflects from reactive centrifugal force. The center of translation corresponds to the inertial reference frame, or static stability axis, or Z–axis of a continuous rotor. Up to the center frequency of the first system critical speed range, applied torque drives "spin" about the neutral axis, while simultaneously centrifugal force drives lateral orbital translation of the mass axis about this neutral spin axis.

1.2.3 Synchronous whirl:

Above the first system critical speed region, the mass axis has inertially self-centered in space and it is in pseudo-static equilibrium between two bearings. Synchronous whirl is a rotating motion in at super-harmonic frequencies at which the rotor journal neutral axis (about which torque is still applied, and about which rotor spin is centered) orbits about the rotor mass axis. This whirl applies to the neutral axis of each modal element, while the shared inner node is pinned at the rotor COM. On rotors behaving as rigid, this motion incorporates a conical "pivoting" of the journals and the rotor neutral axis about the rotor's COM, with reaction forces proportional to rotating frequency and at constant phase angle referenced to Newtonian coordinates. (This is observed on a Bode plot as an up-sloping amplitude trend vs. speed while at a near-constant phase angle, above the first critical speed region.) On more flexible rotors, vibration displacement and relative phase angle are proportional to rotor rotating frequency, and "phase lag" measured at the journals is dependent on the system dynamic stiffness [9].

1.3. Definition of vibration displacement over the machine operating speed range:

Vibration displacement is defined as the magnitude of the radius of lateral precession or orbit of the rotor relative to the position in space of its pseudo-static rotation centerline. However, vibration displacement should always be interpreted in conjunction with the rotor speed range at the time of the measurement, since an amplitude and phase measurement taken at sub- or super-harmonic frequencies, respectively, is measured relative to a different pseudo-static frame of reference. The progression of rotor modes to be described are shown in Figure 2.

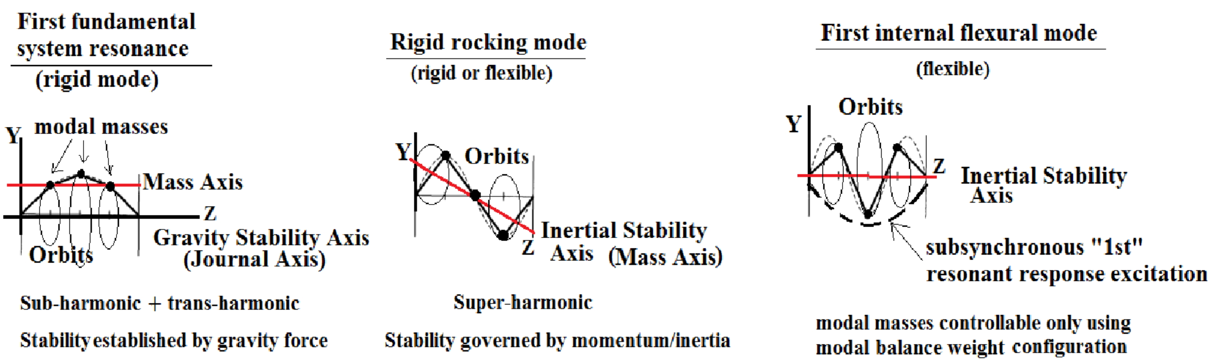


Figure 2. Rotor response modes, orbits and stability axis with increasing speed.

1.3.1. From a state of rest to the initial phase shift of the 1st critical speed range:

The initial position of the journal axis of the rotor in a state of rest corresponds to an inertial reference frame in Newtonian coordinates. Applied torque spins the rotor about this forced, non-centroidal axis as the rotor is accelerated from its state of rest, and its true center of mass axis, being inherently radially offset and/or axially skewed to this axis of rotation, precesses (orbits) around the journal axis. The journal axis acts as the "pseudo-static" stability reference frame corresponding to the axis of the journal shaft centerlines (with the SCL path being a Galilean transformation of the inertial reference frame). Here, the mass axis essentially acts as "unbalance" relative to the journal axis reference.

All rotors within this low speed range may be considered to behave as rigid. A truly rigid rotor will transmit all cyclic forces from rotating unbalance/eccentricity forces into its constraints (bearings), with no forces dissipated into rotor bending but only creating internal material stress. A "flexible" rotor will transmit part of these forces into the constraints, and part into an initial bending or deflecting of the rotor "spring" between the constraints. A very flexible rotor, such as some generator rotors, will maintain a static gravity sag curve and will exhibit continuous axial alternating cyclic compression and tension along the longitudinal axis of the rotor as it rotates. (This condition in practice may accelerate insulation wear of copper windings inserted in slots in the rotor body.)

1.3.2. From initial phase shift to phase angle of ~90 degrees of the 1st system critical speed:

When a rotor is accelerated (adding energy to the system) from the beginning of the 1st critical speed range and from ~10° through ~90° response phase angle shift, the rotor's tendency is to revert to a natural rotation centered around its mass axis. However, it is prevented to do so, being held by gravity in its bearings, with its lateral moment of inertia being larger than forward momentum generated by torque. Therefore below 90° phase, the forces from increasing momentum that would otherwise "flip" the rotor to rotate about its mean mass axis are instead transferred into the forced deflecting of the rotor, as these reactive centrifugal forces in the rotor body continue to react against the imposed constraints at the journals. Some of the added energy used to accelerate the rotor from this point is now instead stored in deflecting the rotor "spring" in the form of potential energy as it bends under increasing reactive centrifugal forces. This condition persists up to the peak of the center frequency of the system fundamental harmonic resonance frequency range and at ~90 degrees phase shift. The centrifugal force from any net-eccentric mass (best observed in a rotating reference frame, oriented by the instantaneous location of the net-eccentric mass) is converted into bending deflection of the flexible rotor "spring", as the entire rotor body laterally translates (orbits) about the journal axis. For very rigid rotors on flexible supports, the transition from 0° to 180° phase is done practically instantaneously. [6]

1.3.3. At ~ 90 degrees of phase angle, and up to 180 degrees:

As a flexible rotor is further accelerated (energy added to the system), upon reaching a phase angle of ~ 90°, the force vectors resisting the rotor from achieving centroidal rotation [6], and the forward momentum are equal in magnitude, and with further sufficient torque to pass this point, the rotor reverts to its "natural rotation", where the axis of rotation (or rotor neutral axis, or axis of spin, about which torque is applied) begins to itself laterally translate around the rotors mean longitudinal mass axis. This can be pictured as the spinning journal centers whirling synchronously around the mass axis. The mass axis now takes the position of becoming the inertial reference frame or "static" stability reference axis, and the overall axial center of mass of the rotor further becomes an inertial constraint in "space" (about which the rotor now pivots) [4], as much as bearing clearances allow. (If drive power is insufficient and unable to generate enough acceleration torque, the rotor can get “stuck” at 90° phase, with amplitude increasing rapidly and risking destruction of the machine).

It should be noted that this behavior applies particularly on a balancing machine with both rotor ends unconstrained. In the field, although this behavior tendency still exists, the added constraints of bearings or adjacent coupled rotors may force the rotation to remain centered about the journal axis, or to only partially achieve this "natural" motion, in which case the constraint to the natural motion will generate high potential energy forces (cyclic stress) upon the supports, and will manifest as "high vibration", generally in the form of bearing seismic vibration, though it can also induce associated kinetic whirling in other unconstrained locations.

As the relative phase angle between the centrifugal force (best recognized in a rotating reference frame, being tied to the net-eccentric mass of the rotating rotor; i.e. the "heavy spot") and the rotor deflection (observed in an inertial reference frame by sensors at the bearings; i.e. the "high spot") becomes greater than 90 degrees, the rotor is accelerated both by the additional torque and in part from the conversion of the potential "spring" energy stored from accelerating the rotor from 0° to ~ 90° phase being converted to kinetic energy as the "spring" deflection releases. This release progresses over the remainder of the phase shift from ~90° to 180°, in accordance with the conservation of angular momentum as the radius of the translational orbit decreases.

1.3.4. Above the 1st critical speed and 180 degrees of phase shift, and up to operating speed above the 1st critical speed frequency, or super-harmonic frequency range in an inertia-governed environment:

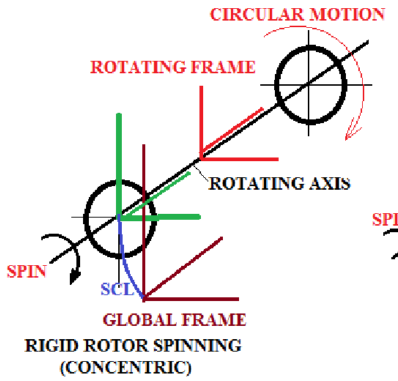
At a phase angle of 180°, the rotor reaches a self-regulated motion between centrifugal forces from any "residual eccentricities" and the residual lateral translation of the rotor journals, i.e. synchronous whirling around the rotor's mass axis (which is now the inertial reference frame or Z-axis in "space"). At this speed, the residual measured displacement amplitudes at the journals indicate the residual mass axis eccentricity of the modal elements of the fundamental harmonic resonance frequency.

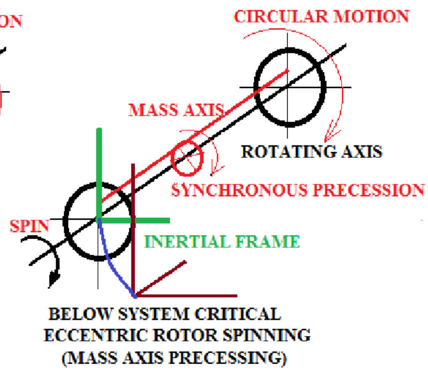
“Vibration displacement” is now referenced as the magnitude of radial displacement of the journals synchronously whirling about the SCL. Each of the two first-mode modal elements is inertially constrained at the COM on one end (the rotor midplane), and elastically connected at the respective bearing supports. The "static" stability orientation of the rotor is now inertially governed relative to the rotor mass axis and controlled by momentum generated from mass eccentricities, with the mass axis self-centered, as far as bearing clearances and seal clearances allow for unconstrained whirling of the journals. The rotor mass axis is now the corresponding Z-axis of the non-rotating X-Y inertial coordinate reference frame, as seen in the SCL, representing rotor static stability in “space”. Residual unresolved eccentricity in each respective modal element that is axially asymmetric to the full rotor COM creates an axial moment about the total rotor COM inertial constraint (a node point), driving its "rocking mode" or whirling of each journal, with the rotor ends out of phase to each other. Each first-mode model element (rotor half) acts as a cantilever constrained at the rotor COM and respective journal, and if excited at higher speeds, each rotor half or modal element individually progresses through its own self-similar process of a “first critical”, out of phase to each other. This is the typical behavior behind what is commonly referred to as "second-critical" vibration displacement.

During further rotor acceleration to operating speed, any residual mass eccentricities present will result in sectional synchronous "whirling", due to axial moments from the unresolved eccentricities and applied torque, proportional to the magnitude of the energy added to the system as an increase in speed or the system load (torque). If at the same time any rubbing occurs between the rotor and case or stator, other higher harmonic frequency responses may be seen in frequency spectrum data plots.

The appearance of any sub-synchronous or super-synchronous frequencies in orbital motion indicates the existence of additional external forces acting on the rotor. These external forces cause subtle cyclic variation of angular velocity evident as a net pulsating torque, which is superimposed upon the basic torque necessary to overcome the total rotor inertia in rotation. This superimposed supplemental "pulsating" torque could excite a response at the rotor's internal natural fundamental harmonic resonance when the rotor itself is at a speed above twice the frequency of the rotor’s fundamental harmonic resonance frequency in free space, destabilizing the static equilibrium of the system. [7, 8]

In summary, the states and reference frames of a rotor as it is accelerated from a state of rest through the system fundamental harmonic resonance frequency and to operating speed are presented in Figures 3a, b, c, d and e.





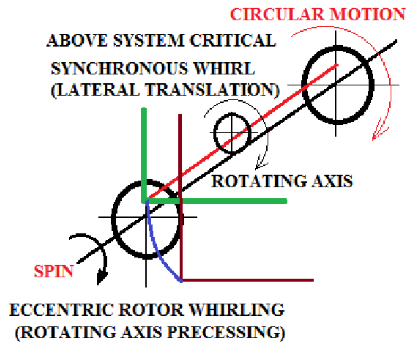
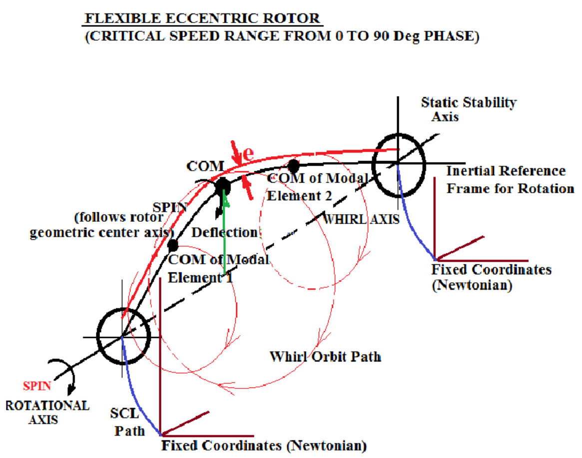


Figure 3a. Concentric rotor. Figure 3b. Mass axis precessing. Figure 3c. Journal axis whirling



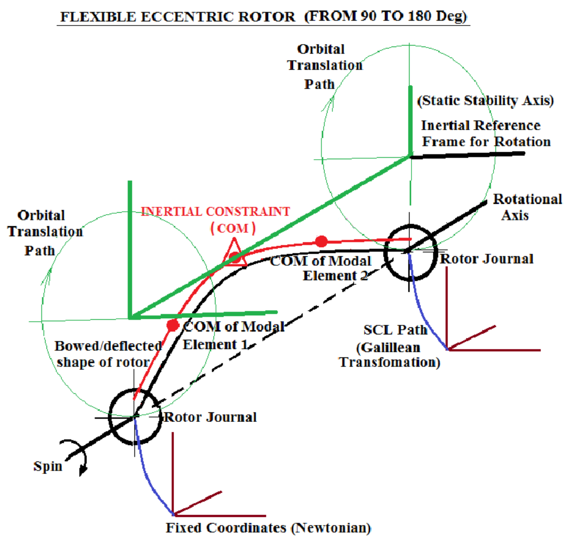


Figure 3d. Flexible rotor mass axis precessing. Figure 3e. Flexible rotor with journals pivoting.

1.4. Hysteresis in shaft centerline path:

When rotor mass axis eccentricities exceed the allowable limit per ISO 1940-1 [1], hysteresis will usually appear in the indicated SCL path, between the path of the rotor while accelerating under drive torque rolling up, and the path when decelerating, rolling down by inertia without drive torque [13]. This is observable both when a rotor is spun solo in a balancing facility, or when operating coupled to other rotors in an operating machine. It can also be seen proportional to increasing/decreasing load torque at constant speed.

This too is a function of inertia-governed "natural" orientation of the rotor. During initial acceleration, as previously described, the rotor will follow an SCL path based on gravity-constrained journals up to the first system critical speed, and based on mass axis orientation at higher speeds. In the case of an assembled rotor train with rotor(s) bowed or with angularly misaligned couplings, there is an additional factor of torque-driven self-straightening. During deceleration without torque, the rotor will maintain the "static" stability orientation governed by its mass-axis for nearly all of the total speed range while decelerating back to the state of rest. This makes the hysteresis observed between the SCL path from the state of rest to running speed and back to a state of rest a good indication of the presence of unresolved mass eccentricity in an individual rotor or assembled rotor train. The magnitude of hysteresis observed in the SCL path is usually proportional to the magnitude of rotor mass eccentricity providing an important diagnostic clue to the source of observed rotor vibration.

2. Vibration in an assembled and coupled rotor train:

When a rotor is assembled on bearings in operation in the field and coupled to other rotors of different size and mass, it is important to consider the effect of the inertia of adjacent rotors, as well as eccentricity induced by misalignment or operational changes in the alignment of bearings and couplings. These induced rotor-train eccentricities can create the same dynamic behavior and altered "natural rotation" tendency as described for distributed mass eccentricities on individual rotors above the first system critical speed range, only now with reference to the mean mass axis of the complete rotor train (across rigid couplings). For each individual rotor in the rotor train, the Z-axis of its own inertial reference frame, representing the rotor's static stability orientation in its bearings when balanced solo in a balancing machine, can also be disturbed in the field when the respective mass axes across coupled rotors are radially offset or become angularly misaligned at the couplings. This misalignment can be caused statically by defective couplings exceeding design machining tolerances or by horizontal misalignment in bearing position. Misalignment can be caused operationally by thermal rise of individual bearings (pedestals or supports) or horizontal looseness, rotors bowing in operation, asymmetric radial pressures in seals, and asymmetric torque from axial operating media mass flow.

In a multi-rotor train, the rotor with the largest mass and inertia will act to govern and influence the "static" stability orientation of other coupled rotors with less mass and inertia. This is most often evident in the "forced" SCL path of outboard journals of a lighter-weight rotor in a rotor train with some angular coupling misalignment, and is seen especially when the machine is operating above the lowest system fundamental harmonic resonance frequency of the rotor train, above the speed at which 180 degrees of phase shift was reached.

The associated observed “vibration” arises when the intended alignment of the lighter rotor is pushed and skewed by and based on the operating alignment of the heavier rotor, seen as SCL motion at the outboard end of the lighter rotor, representing the shift of "static" position/orientation of that rotor's mass axis. If the outboard end of the lighter rotor has sufficient bearing clearance to allow its "static" stability axis into its altered and skewed "natural" inertia-governed orientation, vibration can be exhibited primarily as kinetic displacement in the form of unconstrained conical whirling of the outboard journal. If bearing position or bearing clearance does not allow this inertia-driven reorientation in space and instead acts as a constraint against it, then kinetic displacement may be low but the resulting seismic vibration of the bearing/pedestal may be high, to the extent that the resultant forces are not otherwise absorbed by rotor shaft bending, depending on the rotor's flexibility and bearing damping. (If rotor stiffness is higher than oil film stiffness, the bearing will be wiped.)

Additionally, drive torque further creates an inertial self-straightening of any angular misalignment in the rotor train. The resultant "static" orientation of the rotors during this self-straightening is generally governed by the existing installed orientation of the heaviest rotor in the rotor train, with vibration similarly arising from constraint forces applied against the rotor by any outboard bearing if misalignment is present. While at rated operating speed, the amount of force in the system to drive this vibration is further proportional to MW load and the correlated higher drive torque on the rotor train. This misalignment-caused self-straightening is the cause of many load-proportional increases in vibration, as well as the straight-line SCL shifts sometimes observed while increasing or cycling unit MW load.

Note again, the previous descriptions apply primarily when the assembled rotor train is operating above the speed of the free fundamental harmonic resonance frequency of the rotor with the largest mass. At that speed, the momentum of that rotor becomes sufficiently large to govern the operating orientation of the entire rotor train and displace the initial forced gravity-controlled "static" stability axis orientation.

3. Rotor natural resonance in free space vs. system critical speed:

It is important to recognize the differences between the "resonant response" or harmonic response of a rotor itself, and the "system critical speed response". As a continuous body supported at two ends in bearings, any rotor has an internal inherent natural resonance, appearing only under excitation as a modal response along the Z-axis at the fundamental resonance frequency. As a free body, the fundamental harmonic resonance frequency of the rotor can get excited with any applied sufficient external force perpendicular to the rotor's longitudinal axis, and will react with a modal response motion in the direction of the applied force (e.g. beam impact test of an unconstrained rotor in free space). As a static, non-rotating object, constrained by gravity at supports with finite distance L between supports, the resonant response is true harmonic oscillating planar "vibration" at the rotor's inherent fundamental natural resonance frequency, with the rotor experiencing true internal bending oscillation within its modal response shape. Resulting response modes are numbered according to the number of observed spatial half waves within the span between the rotor constraints.

The terms "resonant response" or "response frequency" are often arbitrarily used to refer to either a time or spatial domain, and this can create much confusion when defining the attributes of "harmonic resonant response" and "system critical speed response". In the spatial domain, resonant response can relate to the wavelength as the physical distance between peaks of the shape of the modal response wave, while in the time domain it relates to the period or cycles per second (Hz) in which that modal response is oscillating. In oscillating harmonic response, the wavelength and period are inversely proportional. If the spatial wavelength is reduced by half, the period (or response frequency in time) will be twice as fast, or be at the 2nd harmonic of the fundamental modal response.

In a rotor, one must distinguish between the "free" resonant oscillating response and the rotating rotor-bearing system response. Both are commonly modeled or represented as being conceptually interchangeable, defaulting to a model of all response as linear oscillation (under questionable assumptions of equivalence of a linear mass-spring-damper model to a rotating rotor system). The inherent "free", natural, non-rotating resonance will act in this way, as a guitar string in planar oscillation. However, a continuous *rotating* rotor in a rotor-bearing *system* requires a reactive centrifugal force to initiate any kind of bending/deflection, and this force originates in the presence of some unbalance or mass eccentricity in combination with some physical constraint preventing "natural" rotation about the rotor's true center of mass axis. This centrifugal force on the rotating rotor does not create a planar, harmonic oscillating "vibration", but rather a constant elastically deflected "static bow", while the rotor itself is synchronously rotating and laterally precessing in orbital motion while remaining essentially rigid.

Both the "first" and "second" system critical speed response of a rotating rotor are effectively rigid modes of the rotor itself, and with a true oscillatory linear vibration occurring only in the bearing supports, as a counter-reaction to the reactive centrifugal forces generated in the rotor due to forced non-centroidal rotation. The rotor-bearing system reacts in a manner somewhat analogous to the true inherent rotor modal responses, with a lateral motion at the first system critical speed and a pivoting/rocking motion at super-critical speeds (after switching the "observer" reference frame to the mass axis), while the respective frequencies of these responses (in time domain) are dependent on the bearing and support stiffness of the system. The bearing stiffness governs the ability of an eccentric rotor to shift its mode of rotation from a forced non-centroidal rotation to a self-aligned "natural" centroidal rotation about its mean mass axis constrained at the rotor COM in space. As examples, a vertical rotor (zero system stiffness) reverts to "natural" centroidal rotation immediately after being accelerated, while a rotor on a horizontal soft-bearing balancing machine can reach this state at ~100 rpm or less, while the same rotor on a completely rigid-bearing machine (such as ball bearings on firm supports with very high stiffness, with journals effectively "clamped") would not achieve this "natural" rotation until reaching extremely high speeds, above practical operating speed of the rotor, if ever.

When looking at the "system critical speed response", it is central to recognize the presence of the inertial constraint at the rotor COM that develops after the first system critical speed region when entering a rigid rocking mode, created due to the rotation of an axially-asymmetric eccentric rotor self-centering about its true mass axis. This midpoint constraint effectively reduces the spatial wavelength of the "free rotor" resonant response by half, and can initiate a resonant response at a frequency of the 2nd harmonic, provided an excitation source is present upon reaching the necessary rotation frequency, generally in the form of centrifugal forces from residual eccentric mass.

Another way of presenting the "free" rotor response at harmonic and super-harmonic frequency can be considered via an analogy with a resonating guitar string. This is analogous to (but not quite physically representative of) the rotor *system* response up to the peak of the first system critical speed (before the change in rotation reference axis) versus at super-critical speeds. In a "plucked" guitar string of length L and stiffness k constrained at each end, there is linear oscillation in the fundamental mode shape at the first inherent harmonic resonant response frequency, and this response is spatially a half-wave. (For string length L, the spatial wavelength of the full wave of the response would be 2L.) If now another constraint is added (a node point) at the guitar string's midpoint, such as pressing on the fret, the string will now oscillate in a full wave, comprising two half-waves out of phase, one on each string half, and the oscillation period (in Hz) of this response occurs at the 2nd harmonic (2x) of the fundamental natural frequency. The full spatial wavelength of the response is now equal to the length L of the string, and each half, being constrained at the midpoint and one end, acts self-similar (as a half-wave) to the fundamental modal response.

4. Rotating machinery design speed criteria: resonance and "critical speed":

In order to design a rotating machine for maximum efficiency, and to preserve rotor stability in hydrodynamic bearings in operation at any speed, it is necessary to consider rotor dynamic behavior for cases of increased inherent rotor eccentricity. This could arise from a developed shaft bow, or induced in the assembly of multiple rotors by misalignment of rotors' mass axes, or by the action of unsteady pulsating torque from operating medium flow, or rubs. The inherent internal fundamental harmonic "resonant response" frequency of the independent rotor itself in free space (denoted as ω here for "free rotor" resonance) is well known as a function of its internal stiffness and internal mass as shown in Equation (1).

$$\omega_{RES} \propto \sqrt{\frac{K_{int}}{M_{int}}} \ (\mathrm{Hz}) \tag{1}$$

Operation at a speed above 2x the fundamental harmonic frequency of the rotor alone (not system) can excite the rotor's internal modal response, and should be used as a design limit of operating speed. Machine design should also focus on the ratio of rotor stiffness to support stiffness. If support stiffness is lower than rotor stiffness, then the rotor will pass through the fundamental system critical speed region while remaining "rigid", and will be able to self-center to "natural" rotation about its mass axis, and correspondingly re-orient its operating alignment in its bearings. If the bearing/support stiffness is greater than rotor stiffness, then the rotor will be prevented from self-centering to rotation about its mass axis as the journals will be effectively "clamped", and centrifugal force and rotor deflection will grow with the square of speed, until potentially reaching a point of destruction of the machine.

If the rotor is operated at a rotational speed (frequency) greater than twice the free rotor's inherent natural resonant response oscillation frequency, then the rotor can itself "vibrate" in a purely internal resonant response (or first internal flexural mode, in a "W" shape). The first and second system modes are "rigid modes" of the rotor which are governed by the bearing/support stiffness as part of the "system criticals". For this third mode, the modal response is integral to the rotor body itself, since nodal points are now within the rotor body between the bearing constraints, and the rotor resonant frequency and response is not controlled by the bearings. Additionally, the inertial midplane constraint at the rotor COM no longer has an effect once the rotor becomes internally "flexible", as it is only acts within the rotor rigid mode above the first system critical speed. Such a response can be reduced by a modal balance weight distribution, but not prevented, with the best solution in such situations being to increase the rotor stiffness by design to prevent reaching this condition. One such solution is to increase rotor diameter and/or shorten the rotor effective length (L) between bearing constraints (e.g. Kaybob compressor). [14]

As an attempt to summarize the design criteria to avoid instability and excessive vibration and forces, the following "proportionality relation" is presented in Figure 4. The "system critical speed", $\mathbf{\Omega_{syscrit}}$ is in time domain (Hz or rpm or rad/sec), while the right side shows the corresponding formula in "time units" but while incorporating the correction factor for the effect of spatial wavelength. Analogous to pressing the midpoint of a guitar string, upon reaching (and passing) the system critical speed peak (90° phase), the rotor COM becomes a constraint in space in the rocking/pivot mode, causing each half of the rotor to react in a manner of a 2nd harmonic of the "free unconstrained" rotor fundamental modal response (itself a half-wave, in which the full spatial wavelength of that 1st modal response would be 2x the rotor length, 2L). The use of the formula form using tension along with the factor of L and 2L is to note the relation between eigenvector (spatial half-wave modal response) as a full wave, and corresponding eigenfrequency (note, this isn't an equation to calculate **Ω**, but a presentation of proportionality and dependencies involved in the terms used to calculate **Ω)**. This hints at the relation between the operating speed relative to the sub- or super-harmonic speed and the corresponding existence of the inertial constraint at the rotor COM (at super-harmonic speeds, creating two spatial "half-waves"), and the excitation of the "free" modal response of the rotor in either fundamental or 2x harmonic resonance. The summed components in the system critical speed relation $\boldsymbol{\Omega}_{\mathbf{sys\ crit}}$ on the right side are shown under vector notation as reference to the resulting real rotor response, noting that the response governed by these stiffnesses is a combination of a linear component (in the bearings/supports) and a rotational component (in the rotor), with the rotational component further incorporating an axially offset angle (phase angle) between the direction of the force (coming from the rotor in circular motion) and the direction of deflection response (a linear reaction against the supports).

(Rotor natural resonance) **(Constrained rotor resonance at rest)** **(Rotor precession angular velocity at critical)** **Bearing oil film and assembly stiffness** **Bearing support total stiffness**

$$\omega_{res\,(n)} \propto \sqrt{\frac{K_{rot\text{-}int}}{M_{rot\text{-}int}}}\ (\mathrm{Hz}) < \omega_{(c)} \propto \sqrt{\frac{K_{rot\text{-}int}}{M_{modal\text{-}1} + M_{modal\text{-}2}}}\ (\mathrm{Hz}) > \Omega_{syst\ crit} \propto \sqrt{\frac{\frac{\left(\frac{\vec{T}_{rot\text{-}int}}{M_{modal\text{-}1} + M_{modal\text{-}2}}\right)}{L}}{2L} + \frac{\vec{K}_{brg}}{M_{brg}} + \frac{\vec{K}_{sup}}{M_{sup}}}\ (\mathrm{Rad/sec})$$

(where: $M_{rot\text{-}int} = V\,\gamma/g$)

$\omega_{res\,(n)}$ = rotor natural resonance in free state

$K_{rot\text{-}int}$ = rotor internal stiffness (stress)

$M_{rot\text{-}int}$ = rotor internal mass

$\omega_{(c)}$ = constrained rotor by gravity at bearings

$M_{modal\text{-}1} + M_{modal\text{-}2}$ = rotor modal mass

V = rotor volume

γ = rotor specific density

g = gravity constant

$\Omega_{syst\ crit}$ = machine system fundamental harmonic resonance frequency

K_{brg} = bearing oil film and assembly stiffness

K_{sup} = bearing and support total stiffness

$T_{rot\text{-}int}$ = rotor "tension" (strain)

M_{brg} = bearing shell mass

M_{sup} = total bearing support mass

L = rotor fundamental harmonic mode distance between bearings

$2L$ = rotor fundamental harmonic resonance frequency wave length

Figure 4: Dependencies between "free rotor resonance" and "system resonance", rotor span L and "response wavelength", and their interrelation with regard to operating design stability.

The above relation in Figure 4 is not intended as a direct equation or means of calculation, but is meant to concisely summarize the required relation between "free rotor" natural resonance frequency $\boldsymbol{\omega}_{\mathbf{res}}$, the inherent internal resonance frequency of the rotor alone when constrained by gravity in bearings **ω(c)**, and the *system* natural resonance frequency $\boldsymbol{\Omega}_{\mathbf{sys\ crit}}$, while also showing the dependencies and proportionalities within the critical speed of a rotor-bearing system. The "free" rotor natural resonance frequency (as if free in space) will be less than the resonance frequency of the "constrained" rotor, simply due to the shorter effective "free" length L of the rotor once the journals are constrained in bearings.

The rotor can run well as long as the fundamental resonance frequency of the independent rotor in bearings **ω(c)** is greater than the system fundamental resonance frequency, $\boldsymbol{\Omega}_{\mathbf{sys\ crit}}$, which is represented as the summed terms under the square root in the above relation. This sum includes the component **ω(c)**, along with a factor of whether its "free" response is a spatial half-wave (fundamental mode response) or full wave (super-harmonic response). This in turn depends on the ratio of bearing/support stiffness to rotor stiffness. The combination of these two stiffnesses determines the system critical speed relative to the independent constrained rotor fundamental resonance frequency, and dictates whether the rotor has achieved “natural” rotation about its mass axis with an inertial constraint at the rotor’s COM. If the rotor’s internal resonance frequency is less than the system resonance frequency, then it will respond in a half-wave (total wavelength 2L) at its own independent resonance frequency, and may reach very high and damaging bending deflection amplitude. If the rotor’s internal fundamental resonance frequency is higher than the system fundamental resonance frequency, then it will switch to “natural” rotation at a lower speed than its own independent resonant response frequency, and enter a “rigid rocking mode”, with a response within in a full spatial wavelength of L.

When operating at a speed greater than 2x the fundamental natural resonant frequency of the free rotor, the internal flexural mode (“W”-shape response) can be excited. In this mode, the operating stability axis no longer passes through the COMs of modal elements as in the two rigid modes. The rotor orientation reverts again to the journal axis (but as an inertial stability axis), but now with the modal masses internally removed/deflected from this axis, and the residual centrifugal forces from residual eccentric mass on each half of the rotor will drive increased bending deflection and also produce a synchronous cyclic force. This in turn can provide excitation to induce a resonant response at the fundamental harmonic frequency of the free rotor, which then appears as a subsynchronous whirl response at that fundamental response frequency. Therefore, a rotor system design should consider the interrelation between the operating rotation speed of the system and the inherent fundamental natural resonance frequency of the rotor alone in a free state or constrained state. The rotor/bearing system should maintain a condition where the maximum operating speed is limited to a rotation speed equal or less than 2x the rotor’s independent fundamental harmonic resonance frequency in free space, shown in Equation (2) to prevent creating instability and uncontrolled rotor vibration in the rotor’s first flexural mode.

$$\omega_{OPER} \leq 2 \text{ x } \omega_{RES} \tag{2}$$

For design purposes, the bearing plus support stiffness must be such that the system critical speed frequency is below the rotor's own first natural resonance frequency, such that the rotor is able to self-align to its mass axis during acceleration through the first system critical speed during machine operation. This requires soft enough bearings and/or supports beneath the bearings. If the bearing/support stiffness is too high, and the rotor reaches its own natural resonance frequency while still "clamped" in the bearings, the deflection arising from centrifugal force (as a reaction force against the support constraints) will grow exponentially and uncontrolled without the rotor being able to “flip” to its natural rotation about its center of mass axis, and the result is high vibration or rotor destruction. In general, it is nearly impossible to optimize bearings for a machine in which the operating speed exceeds 2x the natural resonance of the independent rotor in a free state, since vibration instability forces then arise from within the rotor, independent of the bearings, and the bearings can only aim to absorb the resulting forces, not control them.

One common solution when such symptoms are observed on high speed rotating machines, for which the ratio of mass (inertia) versus power density is much smaller than on the large turbo machinery discussed in this paper, is to incorporate a squeeze film damper bearing or magnetic bearings. However, contrary to the common assumption that its effectiveness is from adding increased damping, its real success is due to its reduced stiffness and its ability to allow the rotor to align to its "natural" rotation orientation according to its mass axis. Adding a soft support under a bearing will create a similar effect, bringing the combined stiffness below the fundamental natural resonance frequency of the free rotor. When vibration and stability problems are seen, it is not necessarily because of an improper bearing design for that rotor, but unsuitable rotor stiffness for that system stiffness.

5. Considerations related to rotor balancing:

Balancing rotors in horizontal machines should be the process of aligning the rotor’s net principal inertia axis (rotor body mass axis) to coincide with the rotor’s gravity-constrained axis of rotation (journal axis) when the rotor is rotating as a rigid body. This correlates conceptually to directly reducing those forces underlying the observed response motion by creating symmetry about the journal axis, versus reducing only the dynamic response motions that the forces may cause without regard to ensuring symmetry (such as placing a couple to “bend back” the responses at the rotor endplanes) [5]. (This is a common practice in field balancing, but successful only on machines with fairly flexible rotors, and when rotors are designed as fairly rigid, such field balancing is not recommended.) The goal is akin to balancing/compensating measured rotor runout directly as a goal of restoring inherent symmetry, as opposed to reducing the eventual dynamic responses caused by this runout when at higher rotation speeds.

This is sometimes phrased as "balancing rigid modes first" [2], and only subsequently balancing residual modal responses. Since rotors are coupled (and constrained) symmetrically about their journal axis, and since rotors are intended by design to spin symmetrically about their journal centers, the optimal balancing approach should be to view this journal-axis rotation and alignment orientation as the basis for all balancing, and displacements should be minimized relative to this journal axis of the rotor. When this is achieved, the eccentric mass axis and journal axis are coincident (approximately, to the best practical extent possible), and the rotor maintains "natural" centroidal rotation about its journal centers through the full speed range, and the rotor remains balanced at all speeds and in all reference frames.

In order to make such a balancing concept as a technological process fully successful in practice, it must be performed at a rotating speed below the peak of the rotor/bearing first system critical speed, while the rotor is still behaving as a “rigid” body and before it self-aligns its rotation center to its mean mass axis. This can be achieved by utilizing the "Pseudo-High Speed Balancing" method which is based on the observed reaction forces [3, 4], or by the "Quasi-High Speed Balancing" method in 2N+1 balancing planes, which is based on the observed reaction motions or displacements [5]. For smaller, high-speed rotors with too small of tolerances, or in cases where pure balancing is not practical, the alternative is to create a softer support structure to allow the rotor to self-align to its inherent mass axis without constraining this self-orientation in its bearings.

Balancing should generally be done with placement of correction weights simultaneously in a minimum of three axially predetermined balancing planes distributed proportionally to resolve the first system critical speed response, i.e. at the total rotor center of mass, and at the center of mass of the two modal elements of the fundamental harmonic mode placed approximately in the so-called quarter planes, with weight amounts corresponding to the axial distribution of eccentric masses. [5] The end planes typically utilized for a "rigid second mode" should be utilized for refinement balancing using a modal "S" weight configuration (e.g. dealing with generators with eccentric retaining rings or fan hubs). The resultant sum of radial forces and axial moments produced by the added weights (with the rotor in a circular motion) will create a virtual dynamic mass axis that should as close as possible mirror the eccentricity of the inherent mass axis, and effectively bring it to within the allowable eccentricity as per ISO 1940-1 [1]. The geometric sum of the two mass axes will result in vanishing of synchronous cyclic reaction forces and displacements at the points of constraints (journals) in operation. The principle of dividing of rotor into modal elements and associated weight placement is shown in Figure 5.

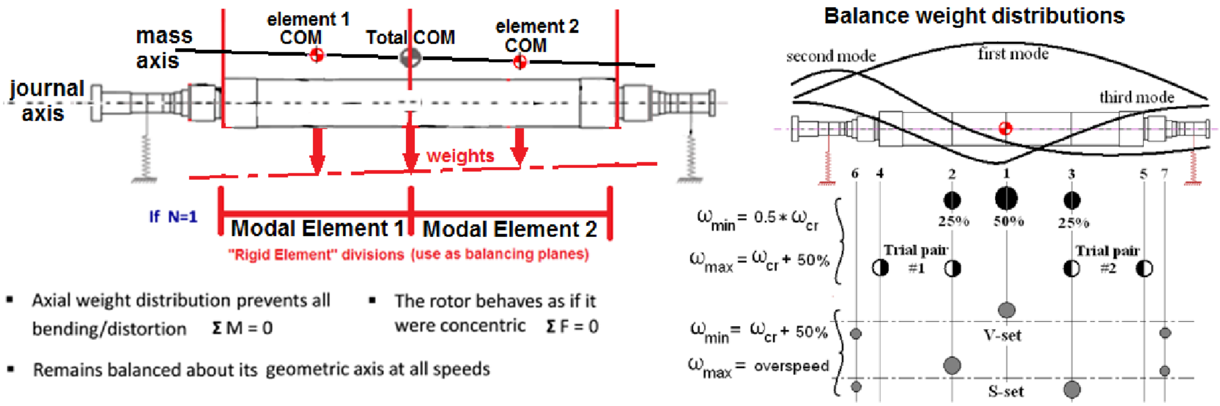


Figure 5. Graphical presentation of division of rotor modal elements relative to modal response, and resulting weight placements in 2N+1 balancing planes.

Once the first critical response is resolved, if vibration amplitudes increase proportionally with increasing speed above the system fundamental harmonic frequency range, or at operating speed, that would indicate that the axial weight distribution should be further optimized. During the balancing process going back to the sub-harmonic frequency range, this can be done using a pair of trial weights per modal element, following the procedures of any balancing method by influence coefficients to fine-tune the axial distribution. However, when refining balancing at operating speed, only modal V or S weight placements must be used, chosen depending on the phase relation from sensors in the same circumferential location, so as not to disturb the radial symmetry gained from the first critical speed solution. If the axial distribution is correct to mirror the distribution of the rotor's inherent mass eccentricity, no "second critical" response should be seen, nor any responses at higher speeds with the machine's operating speed range.

6. Conclusion:

Inertia and momentum-governed behavior of a continuous rotor and a rotor's tendency toward "natural rotation" plays an important role in diagnosing the root cause of rotor vibration problems, and in identifying effective solutions to eliminate undesirable vibration. One must also consider these factors to provide the best balancing results of continuous rotors on balancing machines. This requires a more complex view the real forces and motions in rotating machinery as an open system [11], with recognition of system symmetry with relation to conservation laws [12]. Recognizing the real forces and motions underlying what is perceived as rotor vibration in real operating machines provides important guidelines to rotor-bearing system design, and is integral to optimizing rotor balancing procedures. Furthermore, this understanding is essential to reliably and effectively resolve distributed mass eccentricity on large flexible rotors that operate above their first system critical speed. Rotor balancing should be performed at or up to the "first system critical speed", and should not be only considered as reacting to and "bending back" the specific mode shape deflection at a given speed, but rather should be compensating the mass axis and restoring moment-free symmetry about the journal axis, with the benefit that the rotor will no longer exhibit modal deflections at any critical speeds.

The primary discord in balancing results between standard balancing methods and the Quasi-High Speed Balancing Method developed by the author [6] arises from the fact that standard balancing methods do not consider the switch of axis of the center of rotation, and the corresponding change of constraints which occur when a rotor in a gravitational environment is accelerated through the system fundamental harmonic resonance frequency range. Widely used standard balancing methods work great for rotors with high power density per rotor mass and with minor mass unbalances which do not appreciably change rotor mass axis eccentricity relative to rotating axis, and where residual unbalance forces are absorbed by the bearings as passive device. The advantage of the method developed by the author [6] is that it can deal equally as well as standard balancing methods with rotors with minor unbalances, while providing a very real advantage when balancing large rigid or flexible turbine and generator rotors, and specifically those rotors with bows and "significant" mass axis eccentricities.

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